Practice lesson № 4. Cartesian coordinate system space. The concept of the vector. The projection of a vector by axis. Expansion of the vector in the basis. The scalar and vector product.

Home task: [3] 1-3 page 153, 7-11 page 155, 22-25 page 157, 43-45 page 160, 72-73 page 163, 89-90 page 165.

Examples of solving problems

1. **Determine the length and direction cosines of the radius vector of** $M(4,5,-3)$ **and the vector** $\overrightarrow{M_1M_2}$ **; if** $M_1(3,-2,4)$, $M_2(1,2,3)$

Solution:

a) $\vec{r} = O\vec{M} = (4,5,-3)$; $|\vec{r}| = \sqrt{16+25+9} = 5\sqrt{2}$

$\text{Cos}\alpha = \frac{4}{5\sqrt{2}}$; $\text{Cos}\beta = \frac{5}{5\sqrt{2}}$; $\text{Cos}\gamma = -\frac{3}{5\sqrt{2};}$

б) $\overrightarrow{M_1M_2} = (-2,4,-1)$; $\left|\overrightarrow{M_1M_2}\right| = \sqrt{4+16+1} = \sqrt{21}$

$\text{Cos}\alpha = -\frac{2}{\sqrt{21}}$; $\text{Cos}\beta = \frac{4}{\sqrt{21}}$; $\text{Cos}\gamma = -\frac{1}{\sqrt{21}}$;

$A(2,3,4)$; $B(-2,5,3)$ points are given. Find the coordinates of the point that divides the segment AB in the ratio of

$\lambda = \frac{1}{3}$;

$$X_c = \frac{2+\frac{1}{3}(-2)}{1+\frac{1}{3}} = \frac{(6-2)\cdot 3}{3\cdot 4} = \frac{4}{4} = 1$$

$$Y_c = \frac{\left(3+\frac{1}{3}\cdot 5\right)\cdot 3}{4} = \frac{14\cdot 3}{3\cdot 4} = 3{,}5$$

$$Z_c = \frac{4+\frac{1}{3}}{\frac{4}{3}} = \frac{5\cdot 3}{4} = 3{,}75$$

$C(1,\ 3{,}5,\ 3{,}75)$

2. **Find a job generated by force** $\vec{F}$ **, if its point of application, moving in a straight line, moving from a point** M_1 **to point a** M_2**, i.e.** $\vec{S} = \overrightarrow{M_1M_2}$ **. The work is by the scalar product of the vector** $\vec{F}$ **by the vector** $\vec{S}$ **.**

Решение:

$$A = \vec{F}\cdot\vec{S}$$

$\vec{F} = (2,6,3)$ $\qquad$ $\vec{S} = M_1M_2 = (4,7,3)$

$M_1(-3,-4,2)$ $\qquad$ $A = 2\cdot 4 + 6\cdot 7 + 3\cdot 3 = 8 + 4^2 + 9 = 59$

$M_2(1,3,5)$

3. **vectors:** $\vec{a} = (1,4,6)$; $\vec{b} = (1,1,1)$**, are given.**

Find: a) vector: $5\vec{a} - 2\vec{b}$

б) module: $\vec{a}$

c) $Cos\angle\vec{a}\,\vec{b}$

Solution:

a) $5\vec{a} - 2\vec{b} = 5(1,4,6) - 2(1,1,1) = (5,20,30) - (2,2,2) = (3,18,28)$

б) $|\vec{a}| = \sqrt{1+16+36} = \sqrt{53}$;

в) $Cos\angle\vec{a}\vec{b} = \dfrac{\vec{a}\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{x_1x_2 + y_1y_2 + z_1z_2}{\sqrt{x_{\vec{a}}^2 + y_{\vec{a}}^2 + z_{\vec{a}}^2} \cdot \sqrt{x_{\vec{b}}^2 + y_{\vec{b}}^2 + z_{\vec{b}}^2}} = \dfrac{1\cdot 2 + 4\cdot 1 + 6\cdot 1}{\sqrt{53}\cdot\sqrt{3}} = \dfrac{11}{\sqrt{159}}$;

4. **Find** $(2\vec{a} - 3\vec{b})(\vec{a} - 4\vec{b})$**; if** $|\vec{a}| = 2,\ |\vec{b}| = 4 \qquad \vec{a}\vec{b} = \dfrac{\pi}{3}$;

Solution:

$(2\vec{a} - 3\vec{b})(\vec{a} - 4\vec{b}) = 2\vec{a}^2 - 3\vec{b}\vec{a} - 8\vec{a}\vec{b} + 12\vec{b}^2 = 2\vec{a}^2 - 11\vec{a}\vec{b} + 12\vec{b}^2 =$

$= 2\cdot 4 - 11\cdot 2\cdot 4\cdot Cos\dfrac{\pi}{3} + 12\cdot 16 = 8 - 44 + 192 = 156$

5. **Prove that the vectors** $\vec{a},\vec{b},\vec{c}$ **form a basis. Find the coordinates of the vector** $\vec{d}$ **in this basis, i.e.** $\vec{d} = \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}$**. There is condition for vectors** $\vec{a},\vec{b},\vec{c}$

$$\begin{vmatrix} x_{\vec{a}} & y_{\vec{a}} & z_{\vec{a}} \\ x_{\vec{b}} & y_{\vec{b}} & z_{\vec{b}} \\ x_{\vec{c}} & y_{\vec{c}} & z_{\vec{c}} \end{vmatrix} \neq 0$$

Let

$$\begin{aligned} \vec{a} &= (1,1,2) \\ \vec{b} &= (3,4,1) \\ \vec{c} &= (2,1,3) \\ \vec{d} &= (4,3,1) \end{aligned};$$

Solution:

$$\begin{vmatrix} 1 & 1 & 2 \\ 3 & 4 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 12 + 6 + 2 - 16 - 9 - 1 = -6 \neq 0$$

If $\vec{d} = \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} \Rightarrow \begin{cases} \alpha + 3\beta + 2\gamma = 4 \\ \alpha + 4\beta + \gamma = 3 \\ 2\alpha + \beta + 3\gamma = 1 \end{cases}$ is equate to X, Y, Z vectors $\vec{a}, \vec{b}, \vec{c}$ is equate to vectosr x, y, z. Obtain $\vec{d}$ with help of α, β, γ

$$\begin{cases} \alpha = -3 \\ \beta = 1 \\ \gamma = 2 \end{cases}$$

$$\vec{d} = -3\vec{a} + \vec{b} + 2\vec{c}$$

6. **Calculate the volume of the tetrahedron with vertices**

$A(1, 1, 1);\quad B(2, 1, 0)\quad C(1, 2, 0);\quad D(3, 2, 1)$

Solution:

$$V_{\text{тетр}} = \frac{1}{6} V_{\text{пар-да}} = \frac{1}{6} \overrightarrow{AB} \cdot \overrightarrow{AC} \cdot \overrightarrow{AD} = \frac{1}{6} \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 2 & 1 & 0 \end{vmatrix} = \frac{1}{6}\left(0+0+0+2+0+1\right) =$$

$$= \frac{1}{6} \cdot 3 = 0,5.$$